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COMBINED HEAT TRANSFER IN A BOUNDARY LAYER OF A SELECTIVELY ABSORBING MEDIUM ON A PERMEABLE PLATE UNDER THE CONDITIONS OF INTENSE RADIATION HEATING

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Results of a numerical study of unsteady radiative-convective heat transfer in a boundary layer on a thermally thin permeable plate in the presence of intense radiation heating from outside are reported. The conjugate formulation of the problem takes into account the thermal interaction between the plate and an external gas flow. We consider a turbulent flow of an emitting-absorbing medium with the selective character of absorption. Calculation results are analyzed with a view for clarifying the influence of the governing parameters, namely, the relative temperature of an external radiation source, the Stark number, and the injection parameter. The possibility of inversion of a convective heat flux on the plate under the conditions of high-level external radiation is found.

In the present paper, we report the results of a study dealing with the interaction of a high-temperature gas flow with a solid porous surface through which a cooler is injected. The plate in flow is heated by a gas flow and also by a source of thermal radiation which is external relative to the boundary layer. These conditions are realized in various power installations, the chemical industry, metallurgy, etc.

This model problem is based on an approach developed in [1–4], where unsteady radiative-convective heat transfer in a boundary layer on a thermally thin plate was considered in the conjugate formulation of the problem. The optical properties of a moving medium were assumed to be independent of the radiation frequency (the gray approximation). Here we make an attempt to allow for the selective character of radiation absorption in a gas, which approaches the formulation of the problem to actual situations.

The conjugate problem of radiative-convective heat transfer is considered in a turbulent flow of an emitting-absorbing medium about a thermally thin permeable plate. It is assumed that the optical properties of the medium depend on the temperature and the radiation frequency. The heat capacity of the medium is assumed to be constant, the viscosity and the thermal conductivity are assumed to vary linearly with temperature, and the density is assumed to vary inversely to the temperature. The physical properties of the injected medium are identical to those of the external flow. Radiation transfer along the plate is not taken into account. The injection velocity v_w is taken to be constant over the plate length. Also, it is assumed that the time scale of heating of the boundary layer is much less than that of the plate; this allows one to consider heat transfer quasisteady in the boundary layer. The plate is heated from the initial temperature T_{w0}, and this temperature is maintained constant during the whole period of heating on the plate's site for $0 < x < x_0$. The lower surface and trailing edge of the plate are heat-insulated. Outside the boundary layer, there is a radiation source, which is the black-body surface with temperature T_s. The radiant surface of the source is parallel to the plate.

The thermal state of the plate is described by the nonstationary equation of heat conduction, and the heat transfer in the boundary layer is described by the known set of equations including the equations of

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continuity, motion, and energy. In dimensional form, under appropriate boundary conditions these equations can be found in [4].

Using Dorodnitsyn's transformation, one can solve the dynamic part of the problem irrespective of the thermal part, and, with allowance for the adopted assumptions, it is reduced to the solution of the differential equation

$$((1+\bar{\mu}_t)f'')' + \frac{1}{2}ff'' = \xi\Big(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\Big) \tag{1}$$

with the following boundary conditions when $\eta = 0$: $f = -f_w$ and $f' = 0$ and when $\eta \to \infty$: $f' \to 1$, where f is a dimensionless stream function, $\eta = (\rho_\infty u_\infty/\mu_\infty x)^{1/2}\int_0^y \rho/\rho_\infty\,dy$ and $\xi = x/L$ are the transverse and longitudinal dimensionless coordinates, x and y are the corresponding dimensional coordinates, $f_w = V_w(\mathrm{Re}\,\xi)^{1/2}$, $V_w = \rho_w v_w/\rho_\infty u_\infty$ is the injection parameter, $\mathrm{Re} = \rho_\infty u_\infty L/\mu_\infty$ is the Reynolds number, $\bar{\mu}_t = \mu_t/\mu$ (μ_t is the coefficient of turbulent viscosity), the prime denotes differentiation with respect to the η coordinate, and L is the length of the calculated site of the plate; the subscripts w and ∞ refer to the conditions on the plate and in the external flow, respectively.

The mathematical formulation of the thermal part of the problem in the chosen dimensionless coordinates has the following form:

— heat transfer in the boundary layer

$$\frac{\partial}{\partial\eta}\Big(\Big(\frac{1}{\mathrm{Pr}} + \frac{\bar{\mu}_t}{\mathrm{Pr}_t}\Big)\frac{\partial\theta}{\partial\eta}\Big) + \frac{f}{2}\frac{\partial\theta}{\partial\eta} - \xi f'\frac{\partial\theta}{\partial\xi} - \frac{\mathrm{Sk}}{\mathrm{Re}\,\mathrm{Pr}}\xi\Psi = 0, \quad \xi_0 < \xi < \xi_1, \quad 0 < \eta < \infty; \tag{2}$$

$$\xi = \xi_0:\ \theta = \theta_0, \qquad \eta = 0:\ \theta = \theta_w, \qquad \eta \to \infty:\ \theta \to 1;$$

— heat transfer in the plate in flow

$$\frac{\partial\theta_w}{\partial \mathrm{Fo}} = \frac{\partial^2\theta_w}{\partial\xi^2} - æ\,\mathrm{Sk}\,Q_w, \qquad \xi_0 < \xi < \xi_1, \qquad \mathrm{Fo} > 0; \tag{3}$$

$$\mathrm{Fo} = 0:\ \theta_w = \theta_{w0}, \qquad \xi = \xi_0:\ \theta_w = \theta_{w0}, \qquad \xi = \xi_1:\ \frac{\partial\theta_w}{\partial\xi} = 0.$$

Here $\theta = T/T_\infty$ is the dimensionless temperature, $\theta_0(\eta)$ is the self-similar solution of the energy equation (2) without allowance for radiation, $æ = \lambda_\infty L/\lambda_c H$ is the conjugation parameter, H is the plate thickness; $\mathrm{Fo} = a_c t/L^2$, $\mathrm{Pr} = \mu_\infty/\rho_\infty a_\infty$, and $\mathrm{Sk} = 4\sigma T_\infty^3 L/\lambda_\infty$ are the Fourier, Prandtl, and Stark numbers; Pr_t is the turbulent Prandtl number, a is the thermal diffusivity; $\xi_0 = x_0/L$, $\xi_1 = x_1/L$, x_0, and x_1 are the boundaries of the calculated site of the plate, and the subscript c refers to the plate material.

The expression for the dimensionless divergence of the radiant flux in Eq. (2) has the form

$$\Psi = \int_\Delta \frac{\tau_{vL}(E_{0v} - E_{*v})}{4\sigma T_\infty^4}\,dv, \tag{4}$$

where $E_{0v}(T) = 2\pi h v^3/(c^2(\exp(hv/kT) - 1))$ is the specific black-body radiation flux, $E_{*v} = 2\pi\int_{-1}^{1} I_v(\tau_v,\gamma)\gamma\,d\gamma$ is the density of the incident radiant flux, I_v is the radiation intensity, γ is the cosine of the angle between the ordinate and the direction of the radiation, $\tau_{vL} = \alpha_v L$ is the characteristic optical thickness, α_v is the absorption coefficient of the medium, and the subscript v refers to the spectral quantities. Integration over the frequency in Eq. (4) is performed in the spectral range Δ, in which the medium is opaque. The optical depth in a cut with the ξ coordinate of the boundary layer is expressed as

$$\tau_v = \Big(\frac{\xi}{\mathrm{Re}}\Big)^{1/2}\int_0^\eta \frac{\tau_{vL}}{\theta}\,d\eta$$

and is a function of frequency and temperature.

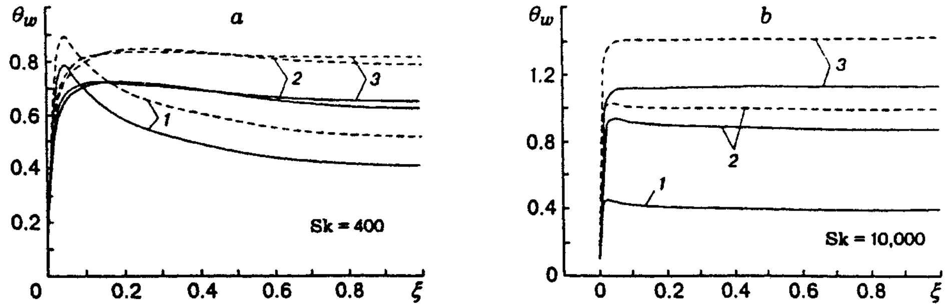

Fig. 1. Dynamics of heating of the plate as a function of the temperature of the source at low (a) and high (b) levels of external radiation: curves 1 and 2 refer to the second and fourth time steps, curves 3 refer to the steady state; (a) $\theta_s = 1.2$ (solid curves) and 1.5 (dashed curves) and $\Delta\mathrm{Fo} = 4 \cdot 10^{-4}$; (b) $\theta_s = 1.2$ and $\Delta\mathrm{Fo} = 5 \cdot 10^{-5}$ (solid curves) and $\theta_s = 1.5$ and $\Delta\mathrm{Fo} = 2.5 \cdot 10^{-5}$ (dashed curves).

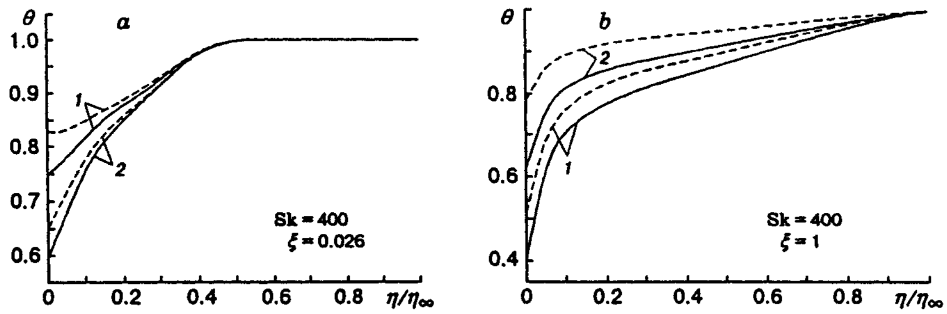

Fig. 2. Dynamics of the temperature field in the gas near (a) and far (b) from the leading edge of the plate as a function of the temperature of the source at a low level of radiation: $\theta_s = 1.2$ (solid curves) and 1.5 (dashed curves) and $\Delta\mathrm{Fo} = 4 \cdot 10^{-4}$; curves 1 refer to the second time step, and curves 2 to the steady state.

The radiant heat transfer in the system considered, which represents a planar layer of an emitting-absorbing medium between the source and plate surfaces, is described by the radiation-transfer equation. To solve this equation, we use the method of average flows [5] within the framework of which the calculation of the radiation field is reduced to the solution of a set of equations

$$\frac{d}{d\tau_v}(\Phi_v^+ - \Phi_v^-) + (m_v^+\Phi_v^+ - m_v^-\Phi_v^-) = \Phi_{0v}(T), \quad \frac{d}{d\tau_v}(m_v^+\delta_v^+\Phi_v^+ - m_v^-\delta_v^-\Phi_v^-) + \Phi_v^+ - \Phi_v^- = 0$$

with the boundary conditions

$$\tau_v = 0: \quad \Phi_v^+(0) = \varepsilon_w\Phi_{0v}(T_w) + (1-\varepsilon_w)\Phi_v^-(0), \qquad \tau_v = \tau_{v\infty}: \quad \Phi_v^-(\tau_{v\infty}) = \Phi_{0v}(T_s).$$

Here $\Phi_v^\pm = 2\pi \int\limits_{0(-1)}^{1(0)} I_v(\tau_v, \gamma)\gamma\, d\gamma / 4\sigma T_\infty^4$ is the dimensionless hemispherical radiation flux, $\Phi_{0v}(T) = E_{0v}(T)/4\sigma T_\infty^4$ is the dimensionless specific black-body radiation flux;

$$m_v^\pm = \int\limits_{0(-1)}^{1(0)} I_v(\tau_v, \gamma)\, d\gamma \Big/ \int\limits_{0(-1)}^{1(0)} I_v(\tau_v, \gamma)\gamma\, d\gamma and \qquad \delta_v^\pm = \int\limits_{0(-1)}^{1(0)} I_v(\tau_v, \gamma)\gamma^2 d\gamma \Big/ \int\limits_{0(-1)}^{1(0)} I_v(\tau_v, \gamma)\, d\gamma$$

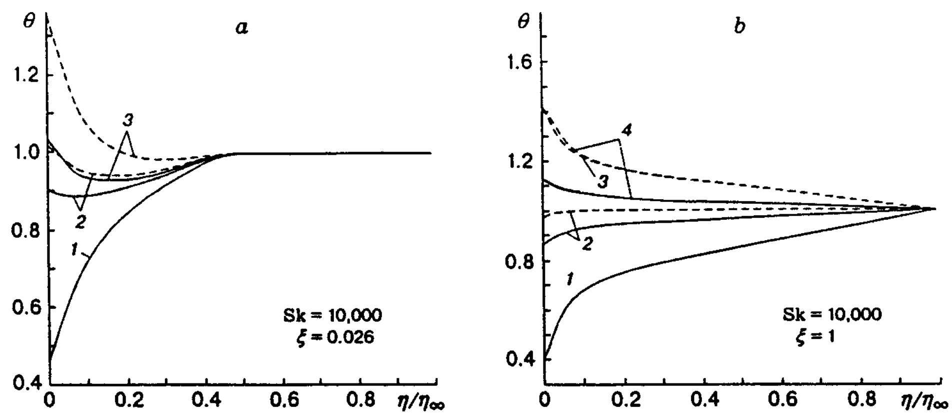

Fig. 3. Dynamics of the temperature field in the gas as a function of the temperature of the source at a high level of radiation: (a) near the leading edge of the plate (curves 1 and 2 refer to the second and third time steps, and curves 3 to the steady state); (b) far from the leading edge of the plate (curves 1, 2, and 3 refer to the second, fourth, and eighth time steps, respectively), $\theta_s = 1.2$ and $\Delta\text{Fo} = 5 \cdot 10^{-5}$ (solid curves) and $\theta_s = 1.5$ and $\Delta\text{Fo} = 2.5 \cdot 10^{-5}$ (dashed curves).

are the transfer coefficients [5], T_s is the temperature of the source, $\tau_{v\infty} = \left(\frac{\xi}{\text{Re}}\right)^{1/2} \int_0^{\eta_\infty} \frac{\tau_{vL}}{\theta} d\eta$ is the optical thickness of the boundary layer, and ε_w is the emissivity of the plate.

In Eq. (3), the total dimensionless specific heat flux on the plate Q_w is determined by the expression

$$Q_w = -\frac{1}{\text{Sk}}\left(\frac{\text{Re}}{\xi}\right)^{1/2} \frac{\partial\theta}{\partial\eta}\bigg|_{\eta=0} + \Phi_w + \frac{\text{Pr}}{\text{Sk}}\left(\frac{\text{Re}}{\xi}\right)^{1/2} f_w\theta_w,$$

where $\Phi_w = E_w/4\sigma T_\infty^4$ and E_w is the specific total resulting radiation flux on the plate.

The Cebeci–Smith two-layer model [6] was applied for calculation of the velocity field in the turbulent boundary layer.

Equation (1) was integrated by the iteration-difference method. The thermal part of the problem was solved by successively refining the temperature of the plate on the basis of a joint solution of the energy and radiation-transfer equations with the boundary condition on the interface which depends on the required temperature. The method of solution was explained in [7].

As the medium to be investigated, we used a mixture of carbon dioxide and an aqueous vapor. The selective character of radiation absorption in a gas was taken into account by the narrow-band method, which is based on Goody's statistical model [8]. It is assumed in this model that the absorption lines are distributed arbitrarily in the spectrum of frequencies, and the line intensity obeys a certain law. The exponential distribution is used most often. Within the framework of this method, the spectral absorption coefficient at low pressures can be represented as follows:

$$\alpha_v = P(k_{v\text{CO}_2}c_{\text{CO}_2} + k_{v\text{H}_2\text{O}}c_{\text{H}_2\text{O}}),$$

where P is the total pressure of the gas, c are the molar concentrations of the mixture components, and k_v is the average line intensity in the absorption band.

As a parameter of the band, k_v is temperature-dependent. We use the values of this parameter from [9–11] in the temperature range of 300–1500 K. In the radiation transfer calculation, the rotational and the 7250, 5331, and 3755-cm^{-1} bands were used for H_2O and the 667 and 3715 cm^{-1} bands for CO_2. To make allowance for broadening effects, Tain [11] recommended additional parameters. In calculating the total radiation fluxes, integration over the spectral variable v was performed with a step of 25 cm^{-1}, which corresponds to the maximum spectral resolution k_v.

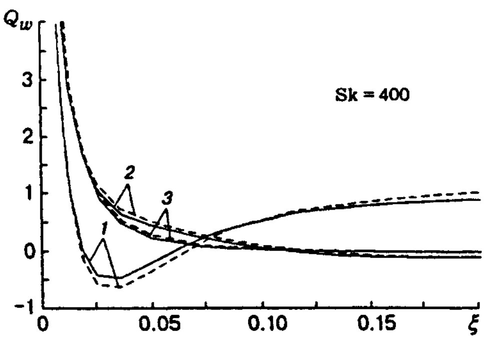

Fig. 4

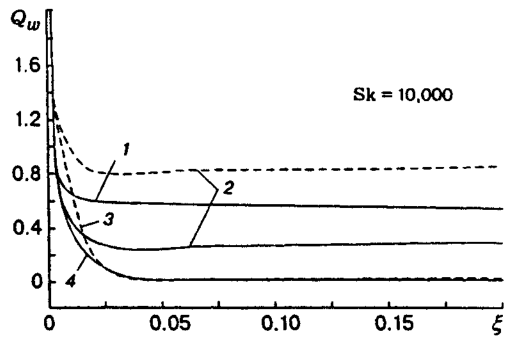

Fig. 5

Fig. 4. Dynamics of the distribution of the total flux Q_w at a low level of radiation: $\theta_s = 1.2$ (solid curves) and 1.5 (dashed curves) and $\Delta\text{Fo} = 4 \cdot 10^{-4}$; curves 1 and 2 refer to the second and fourth time steps, and curves 3 refer to the steady state.

Fig. 5. Dynamics of the distribution of the total flux Q_w along the plate at a high level of radiation: $\theta_s = 1.2$, $\Delta\text{Fo} = 10^{-5}$ (solid curves) and $\theta_s = 1.5$ and $\Delta\text{Fo} = 2.5 \cdot 10^{-5}$ (dashed curves); curves 1, 2, and 3 refer to the second, fourth, and eighth time step, respectively, and curve 4 refers to the steady state.

The results below were obtained for two values of the temperature of the external source $\theta_s = T_s/T_\infty =$ 1.2 and 1.5. This range of variation is due to the restriction from above on the temperature according to the data on the optical properties of the gas.

The calculations were performed for the following values of the governing parameters: $\theta_{w0} = 0.1$, $\text{Pr} = 0.7$, $\text{Pr}_t = 0.9$, $\text{Re} = 10^6$, and $V_w = -0.001$. The emissivity of the surface was taken to be equal to 0.9. It was assumed that $c_{CO_2} = c_{H_2O} = 0.5$. The total gas pressure is $P = 1$ atm.

Figure 1 illustrates the influence of the temperature of the source on the dynamics of heating of the plate at various fractions (levels) of radiation in the total heat flux characterized by the Stark number $\text{Sk} = 4\sigma T_\infty^3 L/\lambda_\infty$. The significant nonmonotony in the distribution θ_w in the initial moments of time, which is associated with convective effects, is noteworthy (Fig. 1a). The higher level of radiation equalizes the plate temperature (Fig. 1b). In the case of a hotter source (for $\theta_s = 1.5$), the temperature curves are located above the corresponding curves for $\theta_s = 1.2$, which is caused by the influence of the radiation flux from the source.

At a high level of radiation transfer (Fig. 1b), the plate is heated almost uniformly over its entire length, and the nonmonotony in the behavior of the curves is almost absent.

Figures 2 and 3 show the dynamics of the temperature field in the boundary layer as a function of the source temperature. An analysis of the behavior of the temperature curves at a low level of radiation transfer (Fig. 2) shows that the gas in the boundary layer has a temperature lower than the temperature of the external flow T_∞. The increase in the temperature of the source T_s (dashed curves) leads only to insignificant heating of the gas.

The temperature curves exhibit an interesting feature at a high level of radiation transfer (Fig. 3). It is seen that in the initial period of heating, in the case of a rather cold source (solid curves 1 and 2 in Fig. 3b) the temperature profile has the standard shape characteristic of conditions under which the temperature of the plate is less than the gas temperature in the external flow. Here convection and radiation are responsible for heating of the plate.

An increase in temperature of the source (dashed curves in Fig. 3) increases the radiation incident on the plate and, as a result, the temperature of the plate can become higher than that of the external flow, i.e., $\theta_w = T_w/T_\infty > 1$. In this situation, the direction of the convective flux changes and the plate heats the gas.

Figures 4 and 5 illustrate the dynamics of the distribution of the total heat flux Q_w along the plate and of the level of radiation transfer in the boundary layer as a function of the temperature of the source.

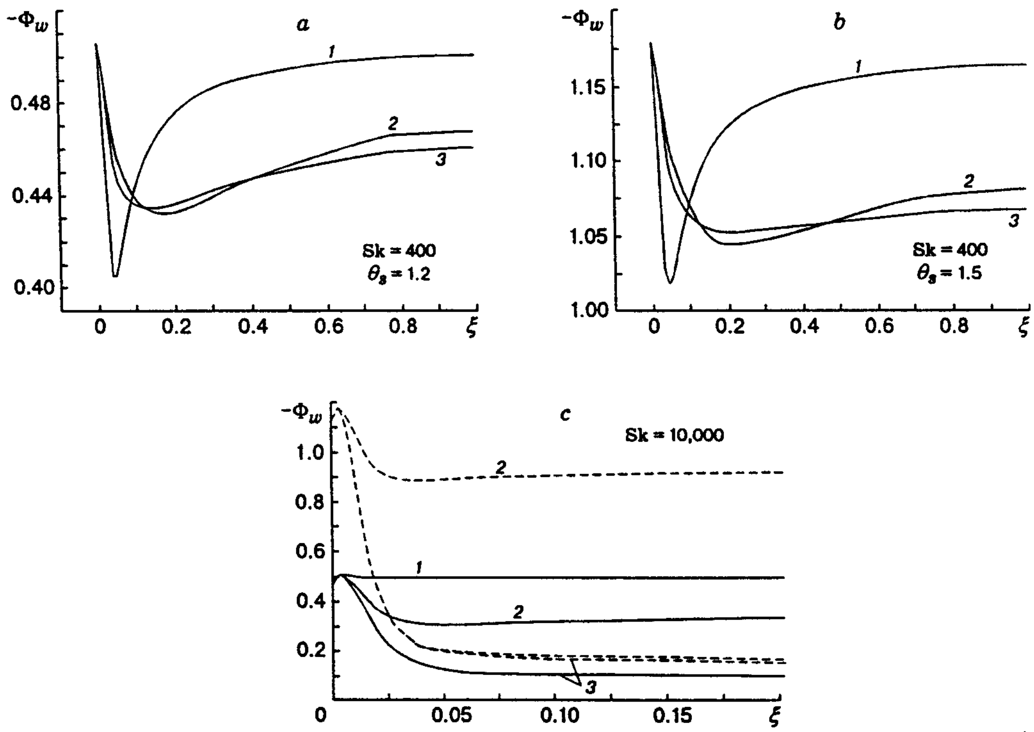

Fig. 6. Dynamics of the distribution of the radiation flux Φ_w at a low ($\Delta\text{Fo} = 4 \cdot 10^{-4}$) (a and b) and a high (c) level of radiation: $\Delta\text{Fo} = 5 \cdot 10^{-5}$ and $\theta_s = 1.2$ (solid curves); $\Delta\text{Fo} = 2.5 \cdot 10^{-5}$ and $\theta_s = 1.5$ (dashed curves); curves 1 and 2 refer to the second and fourth time steps, and curves 3 refer to the steady state.

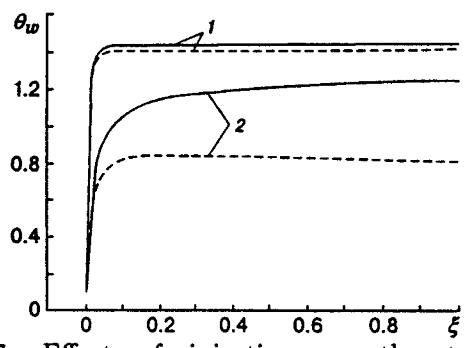

Fig. 7. Effect of injection on the temperature distribution of the plate under steady-state conditions: $V_w = 0$ (solid curves) and 0.001 (dashed curves); Sk = 10,000 (curves 1) and 400 (curves 2); $\theta_s = 1.5$.

Since the lower surface of the plate is heat-insulated in the given formulation of the problem, it means the zero transverse component of the total flux. However, the decrease in Q_w from indefinitely large values as $\xi \to 0$ is caused by the heat flux along the plate because of the presence of the longitudinal temperature gradient in it. With distance from the leading edge, the plate becomes more isothermal, and, therefore, the longitudinal component of the total flux decreases. Clearly, a rise of the temperature of the source (cf. the solid and dashed curves) in stationary conditions increases the total flux owing to the growth of its radiation component. At a high level of radiation transfer (Fig. 5) the distribution of Q_w becomes more uniform, which is connected with the uniformity of the distribution of the radiation source over ξ.

The character of the distribution of the radiation flux Φ_w along the plate (Fig. 6) is predominantly determined by interaction of the radiation incident on the boundary layer from the source and the emitted radiation of the plate. The contribution of the medium to radiation transfer is insignificant because of high transparency of the gas. An increase in Sk markedly increases the temperature of the plate (cf. Fig. 1a and 1b), and hence the resulting radiation transfer decreases.

The distribution curves for the resulting radiation flux along the plate show its strong dependence on T_s, which is caused by an abrupt variation of the emitted radiation of the source with temperature.

The nonmonotone character of the behavior of the Φ_w distribution curves in various moments of time in Fig. 6a and b is connected with the nonmonotone behavior of the temperature of the plate (see Fig. 1a). In the case where the level of radiation transfer is high (Fig. 6c), this phenomenon is not observed.

Figure 7 shows the stationary temperature distribution versus the injection parameter V_w. It is seen that the common tendency toward decreasing the level of temperature during injection is preserved for various values of the Stark number. A similar feature is noted also for the lower values of T_s. For large values of Sk, the injection is less efficient because of the small optical thicknesses of the boundary layer.

Our analysis of the calculation results points to a strong effect of the radiation generated by the source on heat transfer in the boundary layer, which can make the convective flux inverse during radiation heating of the plate.

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